Math 72 4.1 Solving Systems of Linear Equations by Graphing (2x2)

4.2 Solving Systems of Linear Equations by substitution & elimination (2x2)

Objectives

1) Determine if an ordered pair is a solution of a system of linear equations.

2) Solve a system of linear equations using graphing
   - by hand
   - on GC (intersect)

3) Classify a system of linear equations
   - consistent independent
   - consistent dependent
   - inconsistent

   from the graph of the system.

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4) Solve a system of linear equations by substitution (algebra)

5) Solve a system of linear equations by elimination (algebra)

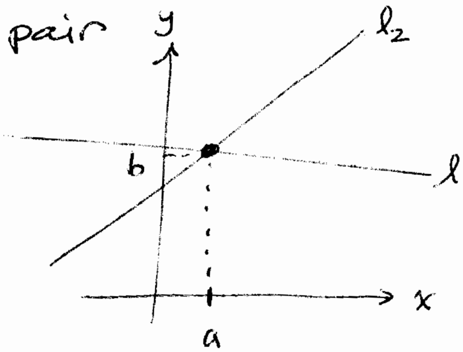
2x2 "two by two" refers to a system having
- 2 equations
- 2 unknown variables

"Solve" vs "Classify" a Linear System

Solve: means to find the values of the variables that make the system of equations true.

(I) • The answer is an ordered pair $\boxed{(a,b)}$

at the point where l_1 intersects l_2

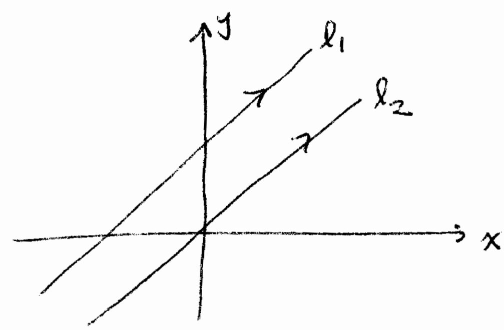


OR

(II) • There is no solution.

$\boxed{\emptyset}$

because l_1 and l_2 are parallel and do not intersect.



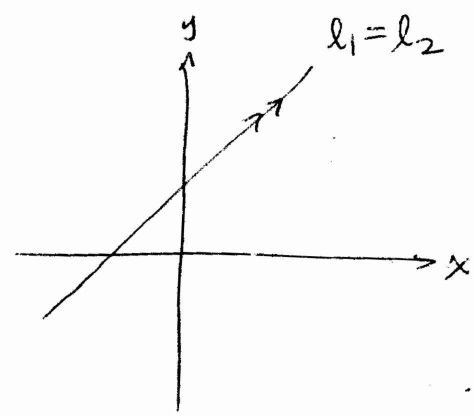
OR

(III) • There are infinitely many solutions

$\{(x,y) : \_\_\_\_\_\_\_\_\_\_ \}$

⇑ write equation of line here

because l_1 and l_2 are the same line when graphed.



Classify: means to identify the characteristics of the system in words.

- consistent: the system has at least one solution.
- inconsistent: the system has no solution

- independent: none of the equations is a combination of the other equations
- dependent: at least one equation is a combination of the other equations.

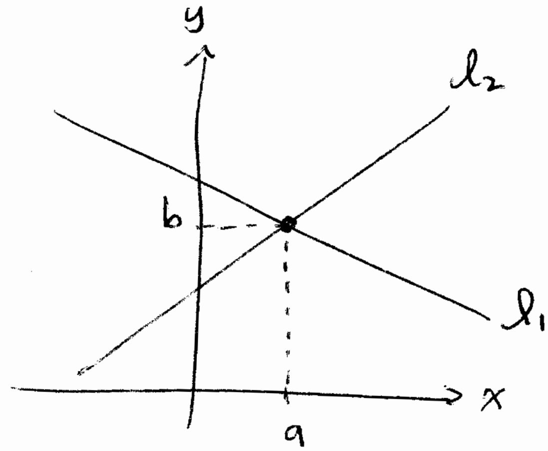
To classify a system (2x2), write two words:

(I) This system has a solution $\Rightarrow$

$\boxed{\text{consistent}}$

The two equations are different.

$\boxed{\text{independent}}$

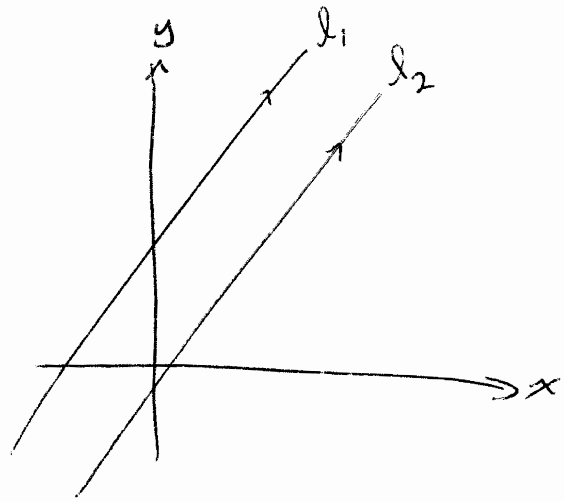


(II) This system has no solution $\Rightarrow$

$\boxed{\text{inconsistent}}$

With 2x2, we don't typically say that the equations are

$\boxed{\text{independent}}$

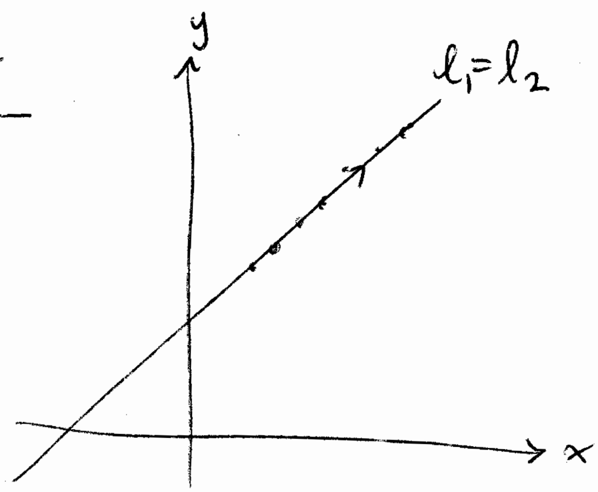


(III) Every point on one line is also on the other line – This system has an infinite number of solutions

$\boxed{\text{consistent}}$

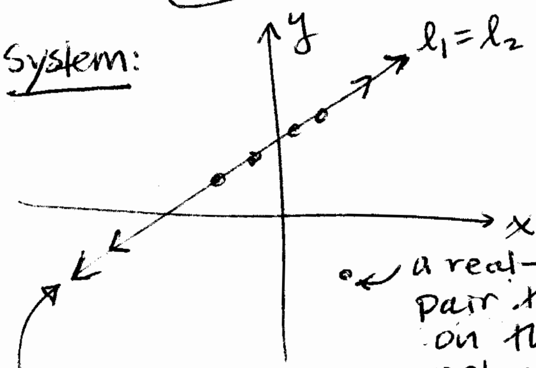
But the two lines are the same line

$\boxed{\text{dependent}}$



CAUTION: When solving a consistent & dependent 2x2 system
Infinitely many solutions (x, y)
is **NOT** "All real numbers x". as for a single equation

System:



solutions must be ON THE LINE to satisfy the equations.

a real-number pair that is not on the line is not a solution. So "all real number pairs" do NOT work in the equations.

solution $\{(x,y): \underline{\quad eqn \quad}\}$

Equation:

solutions on x-axis only.
Any x-value is a solution ⇒ "all real numbers"

$\{x : x \in \mathbb{R}\}$

Quick ① Determine if $(-2.1, .3)$ is a solution of

$$\begin{cases} \frac{-x}{6} + \frac{y}{2} = \frac{1}{2} \\ \frac{x}{3} - \frac{y}{6} = -\frac{3}{4} \end{cases}$$

step 1: substitute $x = -2.1$ and $y = .3$ into 1st eqn (use GC!)

$-(-2.1)/6 + .3/2$

$\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $= \frac{1}{2}$ ✓

>frac

step 2: substitute $x = -2.1$ and $y = .3$ into 2nd eq'n (use GC!)

$-2.1/3 - .3/6$

$\boxed{\text{MATH}}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ $= -3/4$ ✓

step 3: Write yes or no

$\boxed{\text{Yes}}$ $(-2.1, .3)$ makes both equations true.

Useful Observations:

1st: This system can be written with coefficients:

$$\begin{cases} -\frac{1}{6}x + \frac{1}{2}y = \frac{1}{2} \\ \frac{1}{3}x - \frac{1}{6}y = \frac{-3}{4} \end{cases}$$

2nd: We can clear fractions by multiplying each eq'n by its LCD.

$$\left(\frac{-x}{6} + \frac{y}{2} = \frac{1}{2}\right) \text{ by } 6 \Rightarrow \begin{cases} -x + 3y = 3 \\ 4x - 2y = -9 \end{cases}$$

$$\left(\frac{x}{3} - \frac{y}{6} = \frac{-3}{4}\right) \text{ by } 12 \Rightarrow$$

3rd: Soon we'll be able to solve this using matrices instead.

yes ② $\begin{cases} 1.5x + 2.5y = 22.05 & \text{Ⓐ} \\ 3.2x - 0.7y = -15.9 & \text{Ⓑ} \end{cases}$

start @step 2

Solve by graphing on GC.
Approximate solution to nearest hundredth.

Step 1: Isolate y in each equation

Ⓐ $1.5x + 2.5y = 22.05$

$$\frac{2.5y}{2.5} = \frac{-1.5x}{2.5} + \frac{22.05}{2.5}$$

$$y = -0.6x + 8.82$$

Ⓑ $3.2x - 0.7y = -15.9$

$$\frac{-0.7y}{-0.7} = \frac{-3.2x}{-0.7} - \frac{15.9}{-0.7}$$

$$y = \frac{32}{7}x + \frac{159}{7}$$

Use $\boxed{\text{MATH}}$ > frac !!

* CAUTION * Do not use rounded decimals for equation Ⓑ or your answers will include roundoff error

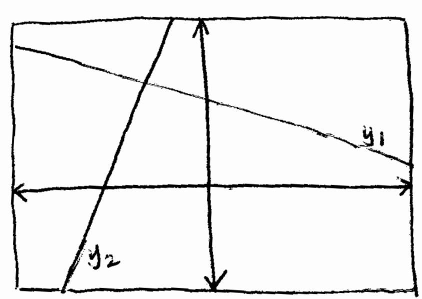
Step 2: Graph both lines in GC.

$y_1 = -0.6x + 8.82$

$y_2 = 32/7 * x + 159/7$

Step 3: Confirm visually that the point of intersection is visible in current GC window. If not, adjust the viewing window.

(YMAX must be MORE THAN 10)



Step 4: Use GC's $\boxed{\text{INTERSECT}}$ command.

$\boxed{\text{2nd}}$ $\boxed{\text{TRACE}}$ = CALC

$\boxed{5}$ = Intersect

$\boxed{\text{ENTER}}$ (1st curve) $\boxed{\text{ENTER}}$ (2nd curve) $\boxed{\text{ENTER}}$ (Guess)

(2) continued

Intersection
X = -2.68674   Y = 10.4322044

<u>Step 5</u>: Round each coordinate to the nearest hundredth. and write an ordered pair.

$\boxed{(-2.69,\ 10.43)}$

classify?
consistent, independent

Yes (3) Solve by substitution

* Review of Method

$$\begin{cases} 2.6x + y = 5.6 & (A) \\ -4.3x - 2y = -4.9 & (B) \end{cases}$$

<u>step 1</u>: Identify best choice of variable to isolate
Isolate y in either equation before choosing x!

Solve (A) for y: $y = -2.6x + 5.6$

<u>step 2</u>: Substitute into (B) to replace y and solve for x.

$$-4.3x - 2(-2.6x + 5.6) = -4.9$$
$$-4.3x + 5.2x - 11.2 = -4.9$$
$$.9x = 6.3$$
$$x = 7$$

<u>step 3</u>: Substitute for x and solve for y.

$$y = -2.6(7) + 5.6$$
$$y = -12.6 = -\frac{63}{5}$$

<u>step 4</u>: Write ordered pair

$\boxed{(7, -12.6)}$ or $\boxed{\left(7, -\frac{63}{5}\right)}$

classify?
consistent, independent

<u>Note</u>: Instructions did not say to round! Give exact answer.

yes (4) Solve $\begin{cases} 3x - 2y = 10 & (A) \\ 4x - 3y = 15 & (B) \end{cases}$ by elimination.

Step 1: Identify best choice of variable to eliminate.

choice #1 eliminate x

$3x$ and $4x$ $\Rightarrow$ the LCM of 3 and 4 is 12

mult (A) by 4: $3x \cdot 4 = 12x$

mult (B) by -3: $4x \cdot (-3) = -12x$

$\overline{0}$

* choice #2: eliminate y

$-2y$ and $-3y$ $\Rightarrow$ the LCM of 2 and 3 is 6.

mult (A) by 3: $-2y \cdot 3 = -6y$

mult (B) by -2: $-3y \cdot (-2) = 6y$

$\overline{0}$

Since 6 is smaller than 12, we choose option 2:

Step 2: Multiply all terms of each equation by multipliers needed to get LCM, with one equation positive and the other negative.

(A) $\times$ 3: $3 \cdot 3x - 3 \cdot 2y = 3 \cdot 10$

$9x - 6y = 30$

(B) $\times (-2)$: $-2 \cdot 4x - (-2) \cdot 3y = (-2) \cdot 15$

$-8x + 6y = -30$

Step 3: Add the two equations together, like terms to like terms: If necessary, isolate variable.

$$\begin{array}{rcrcl} 9x & - & 6y & = & 30 \quad (A) \\ -8x & + & 6y & = & -30 \quad (B) \\ \hline x & & & = & 0 \end{array}$$

Step 4: Substitute result into any previous equation and solve for remaining variable.

subst $x = 0$ into (A):

$$3(0) - 2y = 10$$

$$-2y = 10$$

$$y = -5$$

Step 5: Write solution as an ordered pair.

$$\boxed{(0, -5)}$$

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classify?

consistent, independent

yes 7 Solve and classify.

$$\begin{cases} 3x + \dfrac{y}{2} = 2 & (A) \\ 6x + y = 5 & (B) \end{cases}$$

Method 1: Elimination

$$\begin{cases} 3x + \dfrac{y}{2} = 2 & (A) \\ 6x + y = 5 & (B) \end{cases}$$

← mult by −2 to eliminate x (or to eliminate y)

$$3(-2)x + \frac{y}{2}(-2) = 2(-2) \quad (A) \times (-2)$$

$$\begin{cases} -6x - y = -4 & (A)\text{ new} \\ 6x + y = 5 & (B) \end{cases}$$

add equations
like terms w/ like terms

$$0x + 0x = 1$$

$$0 = 1$$

$0 \neq 1$!?!
False statement ⇒ parallel lines

solve: $\boxed{\text{no solution}}$ or $\boxed{\varnothing}$

classify: $\boxed{\text{inconsistent}}$ or $\boxed{\text{inconsistent independent}}$

Method 2: substitution

Solve (B) for y:

$$\begin{array}{r} 6x + y = 5 \\ -6x \qquad\; -6x \\ \hline y = -6x + 5 \end{array}$$

Substitute into (A):

$$3x + \frac{-6x + 5}{2} = 2$$

$$3x + \frac{-6x}{2} + \frac{5}{2} = 2$$

→

(7) con't $\underbrace{3x - 3x}_{} + \frac{5}{2} = 2$

$$0x + \frac{5}{2} = 2$$

$$\frac{5}{2} = 2 \qquad \frac{5}{2} \neq 2 \text{ !?! } \text{false statement}$$

Solve: $\boxed{\text{no solution}}$ or $\boxed{\phi}$

classify: $\boxed{\text{inconsistent}}$ or $\boxed{\text{inconsistent independent}}$

* When using an algebraic or GC method and our correct work leads to a false statement containing only numbers (no variables), the system has no solution.

(6) Solve and classify.

$$\begin{cases} y = \frac{1}{7}x + 3 & \text{(A)} \\ x - 7y = -21 & \text{(B)} \end{cases}$$

Method 1: Substitution

(A) already has y isolated.

Substitute (A) $\rightarrow$ (B)

$$x - 7\left(\frac{1}{7}x + 3\right) = -21$$

$$x - 7\cdot\frac{1}{7}x - 7\cdot 3 = -21 \qquad \text{distribute}$$

$$\underbrace{x - x}\ - 21 = -21$$

$$0x - 21 = -21$$

$$-21 = -21 \qquad \text{true!?!}$$

solve: $\boxed{\{(x,y): x - 7y = -21\}}$

or $\boxed{\{(x,y): y = \frac{1}{7}x + 3\}}$

Caution: MathXL provides you with a completely filled-in set! But when you take a PQ or exam, you'll have to write it out yourself!!

classify: $\boxed{\text{consistent} \\ \text{dependent}}$

Explore: What does $\{(x,y): y = \frac{1}{7}x + 3\}$ mean?

"The set of all ordered pairs (x,y) so that $y = \frac{1}{7}x + 3$."

example: choose any value of x, plug into eq'n.

$x = 6 \Rightarrow y = \frac{1}{7}(6) + 3 = \frac{27}{7} \Rightarrow \left(6, \frac{27}{7}\right)$ is a solution.

We can repeat for an infinite number of values of x to find many solutions.

(8) con't

Method 2: Elimination

Equation (A) is not in standard form.

$y = \frac{1}{7}x + 3$ (A)

subtract $\frac{1}{7}x$ both sides to get $ax + by = c$

$$\begin{cases} -\frac{1}{7}x + y = 3 & (A) \\ x - 7y = -21 & (B) \end{cases}$$

$-\frac{1}{7} \cdot 7x + 7 \cdot y = 3 \cdot 7$ (A)x7

To eliminate x, multiply equation (A) by 7.

OR To eliminate y, multiply equation (A) by 7.

OR To eliminate x, multiply equation (B) by $\frac{1}{7}$.

OR To eliminate y, multiply equation (B) by $\frac{1}{7}$.

$$\begin{cases} -x + 7y = 21 & (A) \\ x - 7y = -21 & (B) \end{cases}$$

$0x + 0y = 0$ add like terms each side

$0 = 0$ true!?!

solve: $\boxed{\{(x, y) : x - 7y = -21\}}$

or $\boxed{\{(x, y) : y = \frac{1}{7}x + 3\}}$

classify: $\boxed{\text{consistent dependent}}$

Extras:

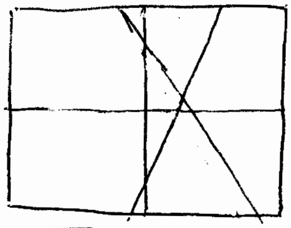
(1) Solve by graphing on GC. Approximate to the nearest hundredth.

$$\begin{cases} y + 2.6x = 5.6 \\ y - 4.3x = -4.9 \end{cases}$$

$$\begin{cases} y_1 = -2.6x + 5.6 \\ y_2 = 4.3x - 4.9 \end{cases}$$

$\boxed{y=}$

$\boxed{\text{ZOOM}}\boxed{6}$



$\boxed{\text{2nd}}\boxed{\text{TRACE}}$ = CALC

5. Intersect

$\boxed{\text{ENTER}}^3$

Intersection
X = 1.5217391
Y = 1.6434783

Round to nearest hundredth

$\boxed{(1.52, 1.64)}$

classify? consistent, independent

(2) Solve the system of equations by the elimination method.

$$\begin{cases} \frac{5}{9}x + \frac{1}{3}y = \frac{34}{3} & \swarrow \text{LCD} = 9 \\ \frac{4}{81}x - \frac{5}{9}y = -\frac{115}{27} & \swarrow \text{LCD} = 81 \end{cases}$$

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

should be → ○A. The solution of the system is ☐.
(Simplify your answer. Type an ordered pair. Use integers or fractions for any numbers in the expression.)

○B. There are infinitely many solutions.

○C. There is no solution.

clear fractions!

$$\begin{cases} \frac{5}{9}x \cdot 9 + \frac{1}{3}y \cdot 9 = \frac{34}{3} \cdot 9 \\ \frac{4}{81}x \cdot 81 - \frac{5}{9}y \cdot 81 = -\frac{115}{27} \cdot 81 \end{cases}$$

$$\begin{cases} 5x + 3y = 102 & (A) \\ 4x - 45y = -345 & (B) \end{cases}$$

elim y by mult (A) by 15

$$\begin{array}{rcl} 75x + 45y &=& 1530 \\ 4x - 45y &=& -345 \\ \hline 79x &=& 1185 \end{array}$$

$$x = 15$$

Subst back to A

$$5(15) + 3y = 102$$
$$3y = 27$$
$$y = 9$$

$$\boxed{(15, 9)}$$

Way Easier: Use GC

MATRIX 2 x 3

$$\begin{bmatrix} 5/9 & 1/3 & 34/3 \\ 4/81 & -5/9 & -115/27 \end{bmatrix}$$

RREF

$$\begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & 9 \end{bmatrix}$$

$$\boxed{(15, 9)}$$

classify?
consistent, independent